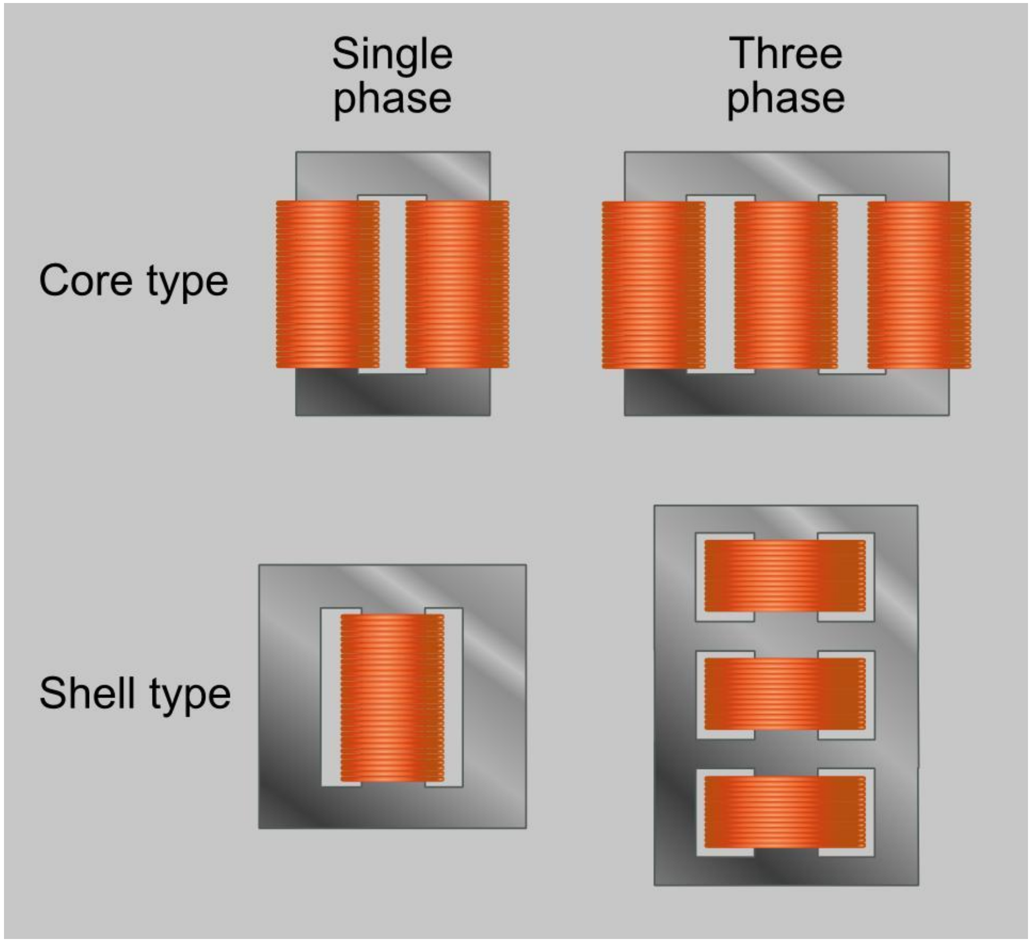
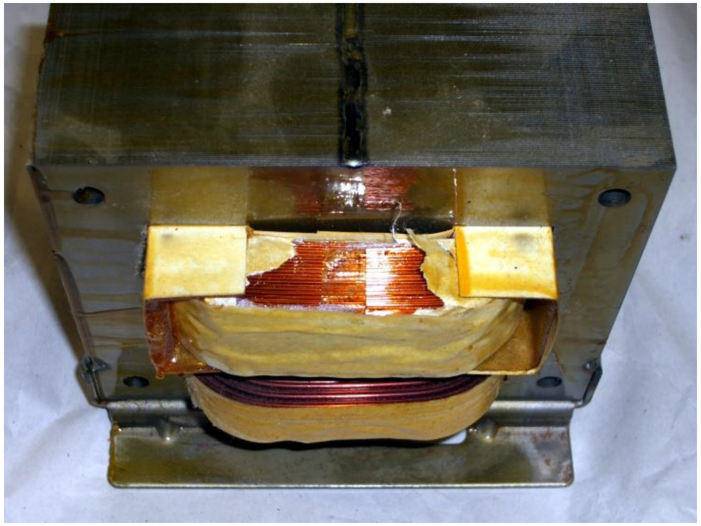
ECG 741
Power Distribution Transformers

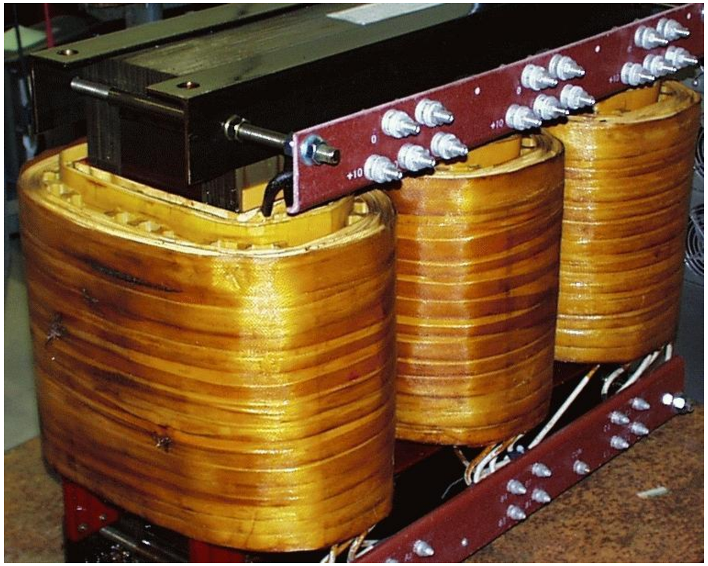
Spring 2014

Preliminary Considerations

A transformer is a device that converts one AC voltage to another AC voltage at the same frequency. The windings are wrapped on top of each other to decrease flux leakage.







Ideal Transformer

An ideal transformer (unlike the real one) can be characterized as follows:

1. The core has no hysteresis nor eddy currents.
2. The magnetization curve is vertical with no saturation
3. The leakage flux in the core is zero.
4. The resistance of the windings is zero.

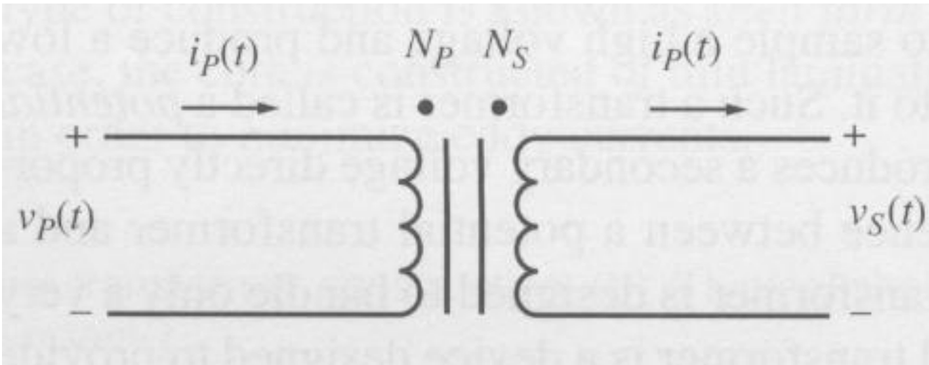
Consider a lossless transformer with an input (primary) winding having N_p turns and an output (secondary) winding of N_s turns.

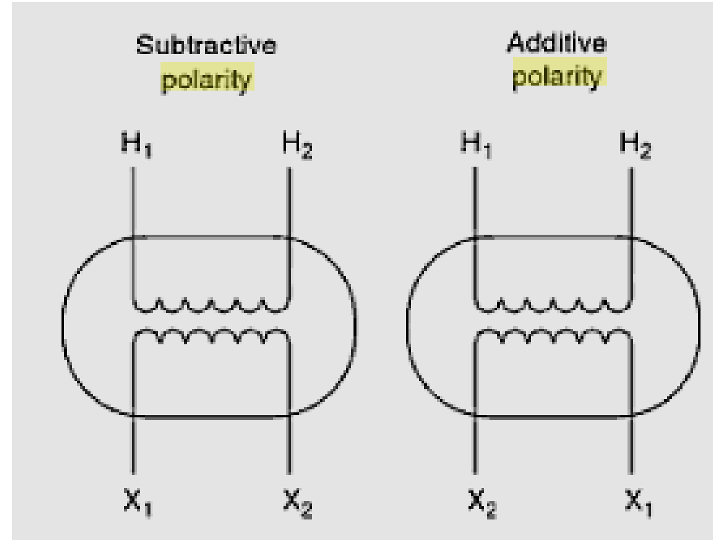
The relationships between the voltage and current applied to the primary winding and the voltage and current produced on the secondary winding are

$$\frac{\mathbf{V}_p}{\mathbf{V}_s} = a \qquad \frac{\mathbf{I}_p}{\mathbf{I}_s} = \frac{1}{a}$$

where $a = N_p/N_s$ is the turn ratio of the transformer.

Lead Markings and Polarity





- Dot markings are used on instrument transformers.
- On power transformers, the terminals are designated by the symbols H1 and H2 for the high-voltage (HV) winding, and by X1 and X2 for the low-voltage (LV) winding.
- By convention, H1 and X1 have the same polarity (which can be additive or subtractive).

Power in an Ideal Transformer

The output power of an ideal transformer equals to its input power – to be expected since assumed no loss. Similarly, for reactive and apparent powers:

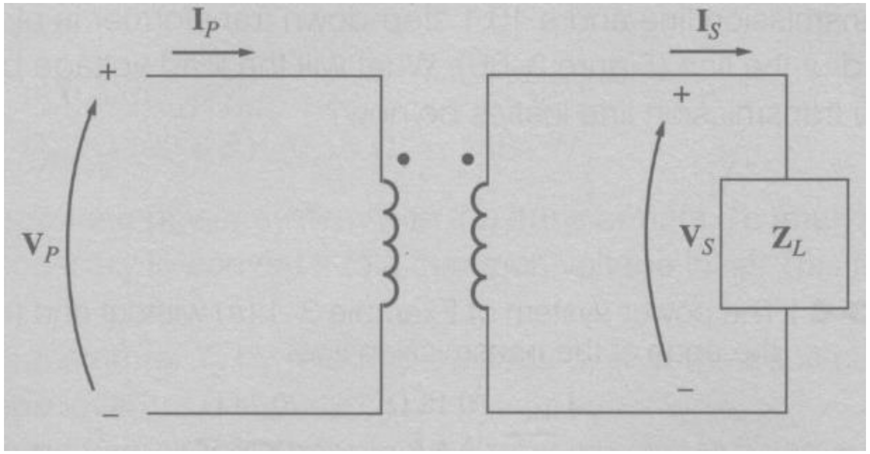
$$P_{out} = V_s I_s \cos\theta = \frac{V_p}{a} a I_p \cos\theta = V_p I_p \cos\theta = P_{in}$$

$$Q_{out} = V_s I_s \sin\theta = V_p I_p \sin\theta = Q_{in}$$

$$S_{out} = V_s I_s = V_p I_p = S_{in}$$

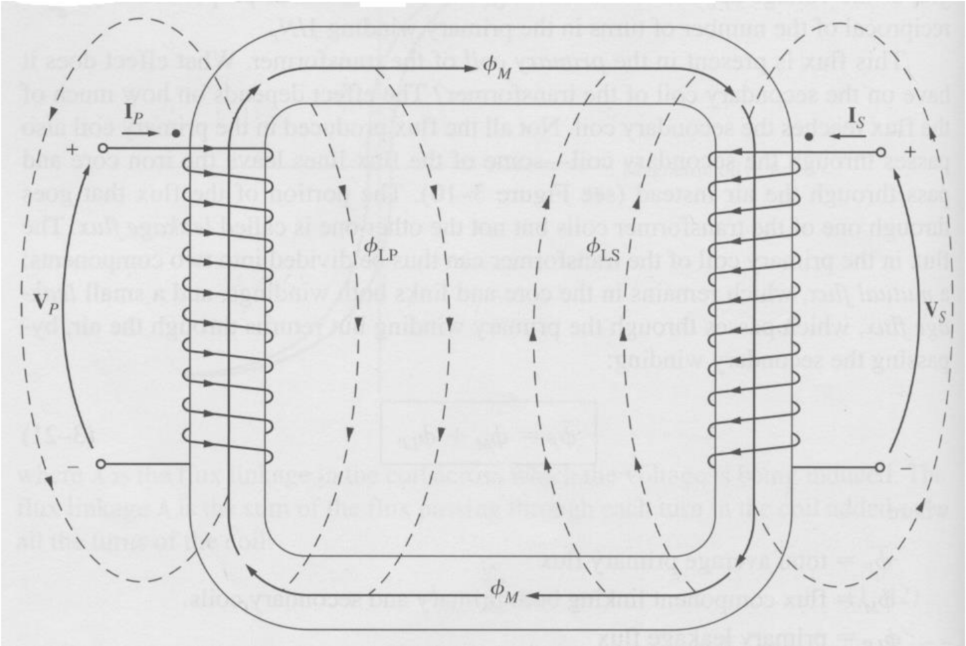
Apparent impedance of the primary circuit:

$$Z_L' = \frac{\mathbf{V}_p}{\mathbf{I}_p} = \frac{a\mathbf{V}_s}{\mathbf{I}_s / a} = a^2 \frac{\mathbf{V}_s}{\mathbf{I}_s} = a^2 Z_L$$



Leakage Flux in Real Transformer

A portion of the flux produced in the primary coil passes through the secondary coil (mutual flux); the rest passes through the external medium (leakage flux):



$$\overline{\phi}_p = \phi_m + \phi_{Lp}$$

mutual flux   leakage primary flux

Similarly, for the secondary coil:

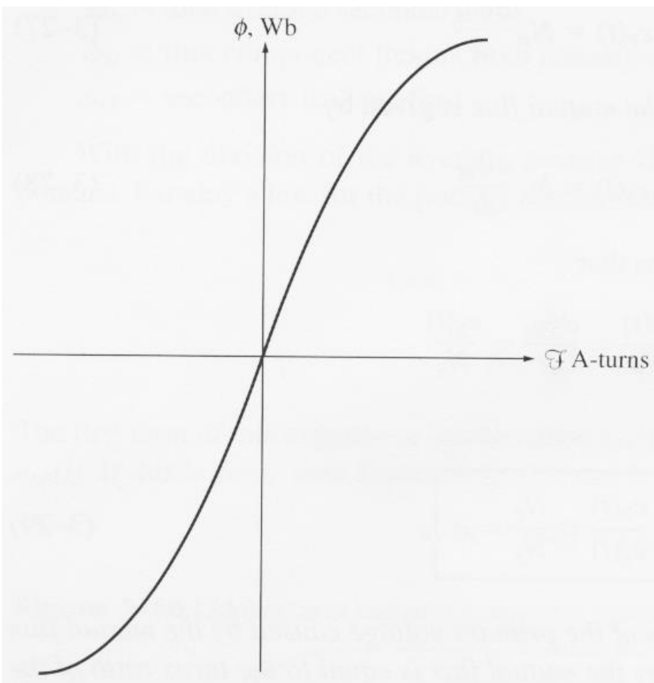
$$\overline{\phi}_s = \phi_m + \phi_{Ls}$$

Leakage secondary flux

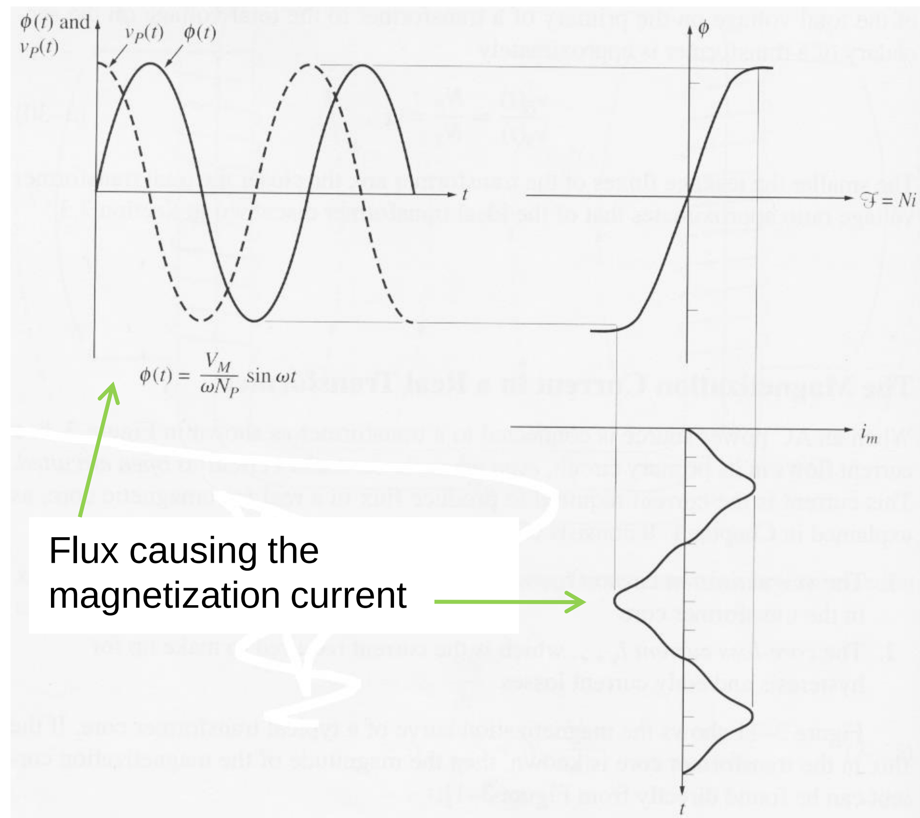
Excitation Current in a Real Transformer

Even when no load is connected to the secondary coil of the transformer, a current will flow in the primary coil. This current consists of:

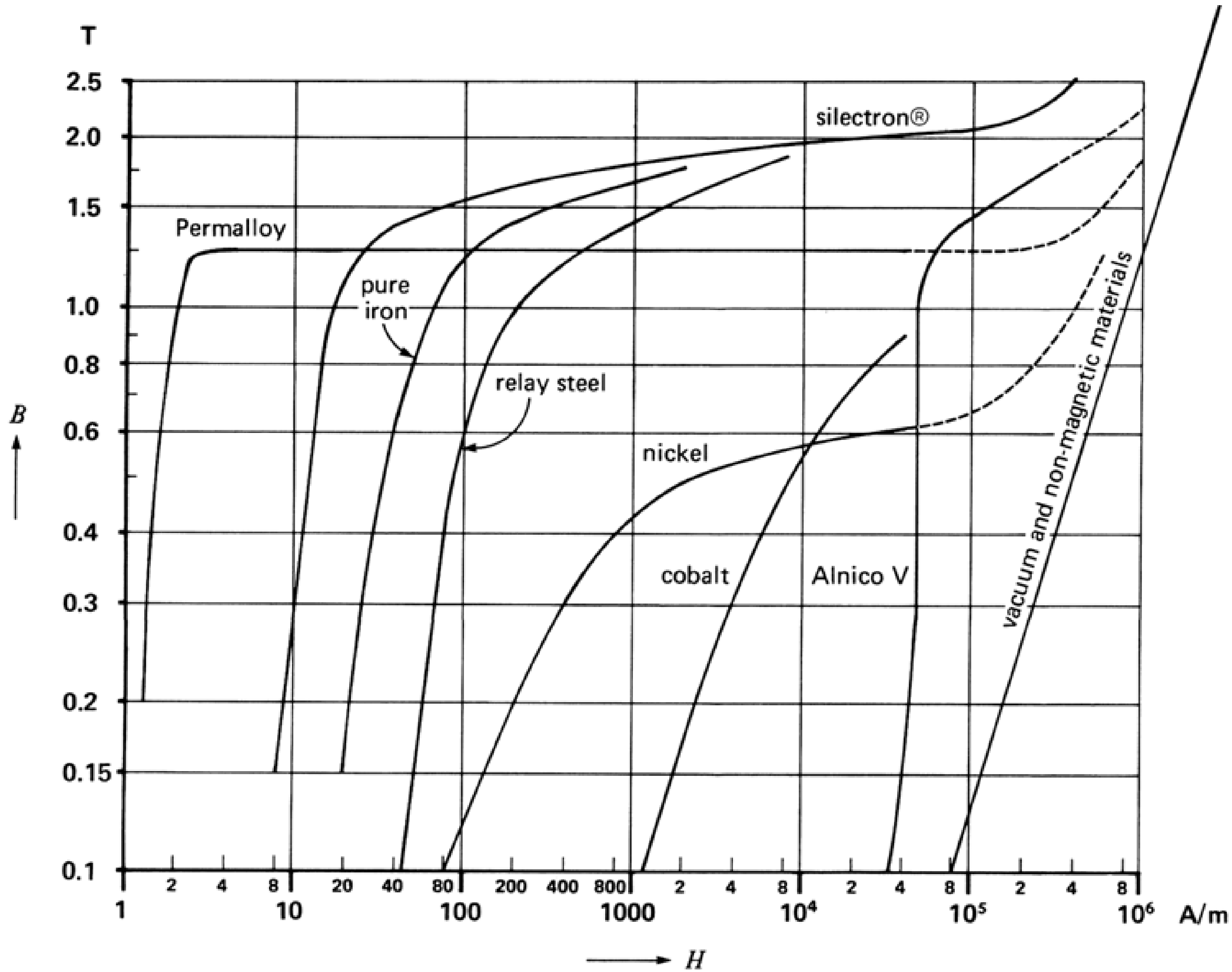
1. Magnetization current i_m is needed to produce the flux in the core;
2. Core-loss current i_{h+e} corresponds to hysteresis and eddy current losses.



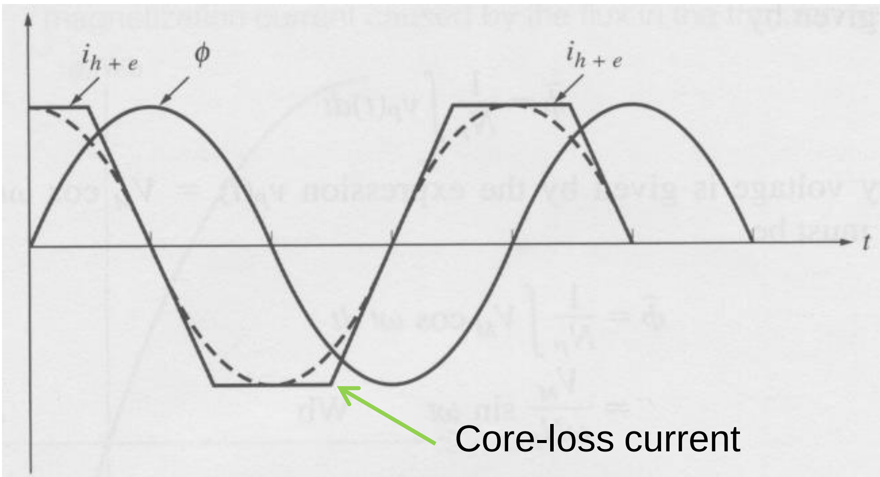
Typical magnetization curve

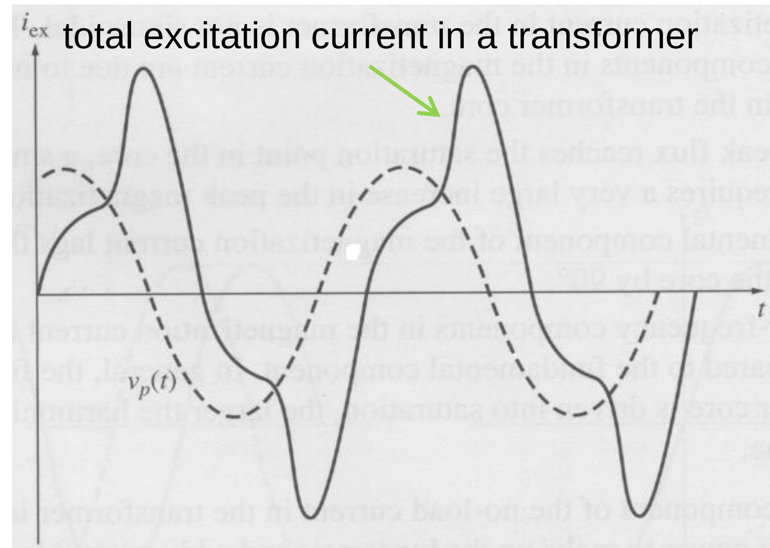


Saturation curves of magnetic and nonmagnetic materials



Excitation Current in a Real Transformer





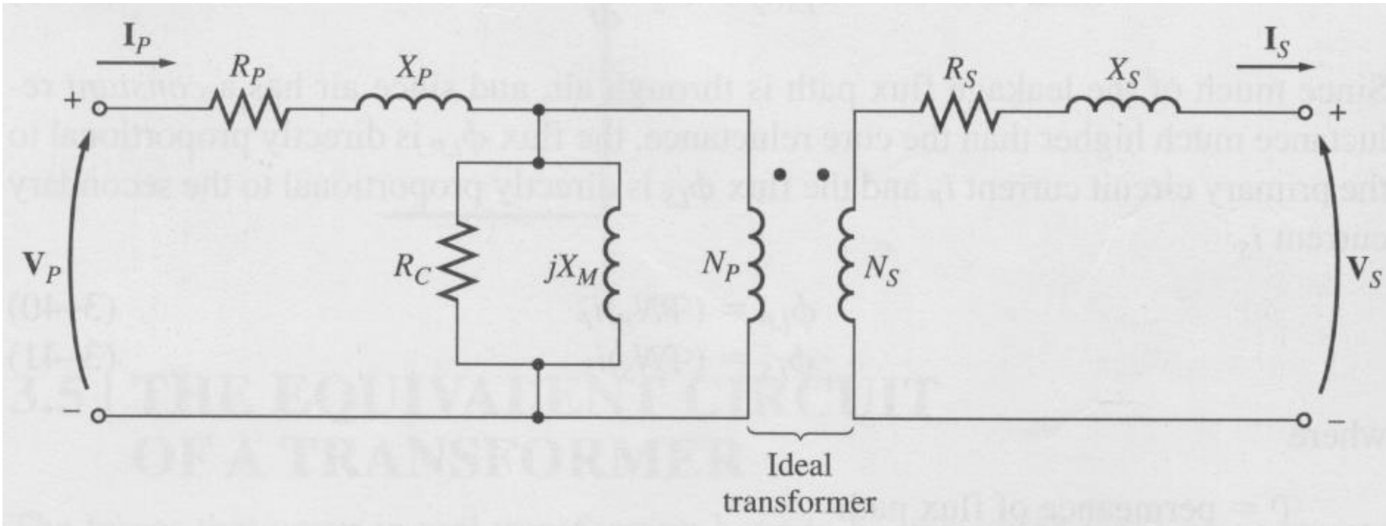
Core-loss current is:

1. Nonlinear due to nonlinear effects of hysteresis;
2. In phase with the voltage.

The total no-load current in the core is called the excitation current of the transformer:

$$i_{ex} = i_m + i_{h+e}$$

Equivalent Circuit of a Real Transformer



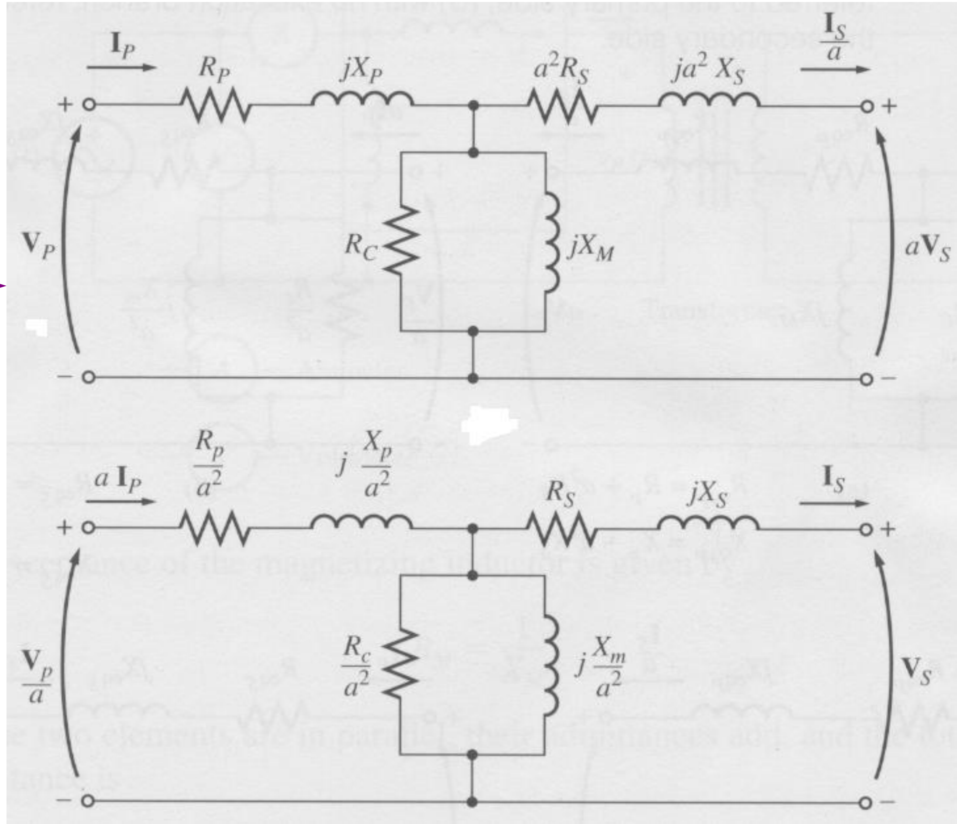
- Cooper losses are modeled by the resistors R_p and R_s.
- The leakage flux can be modeled by primary and secondary inductors.
- The magnetization current can be modeled by a reactance X_M connected across the primary voltage source.
- The core-loss current can be modeled by a resistance R_C connected across the primary voltage source.
- Both magnetizing and core loss currents are nonlinear; therefore, X_M and R_C are just approximations.

Equivalent Circuit of a Real Transformer

The equivalent circuit is usually referred to the primary side or the secondary side of the transformer.

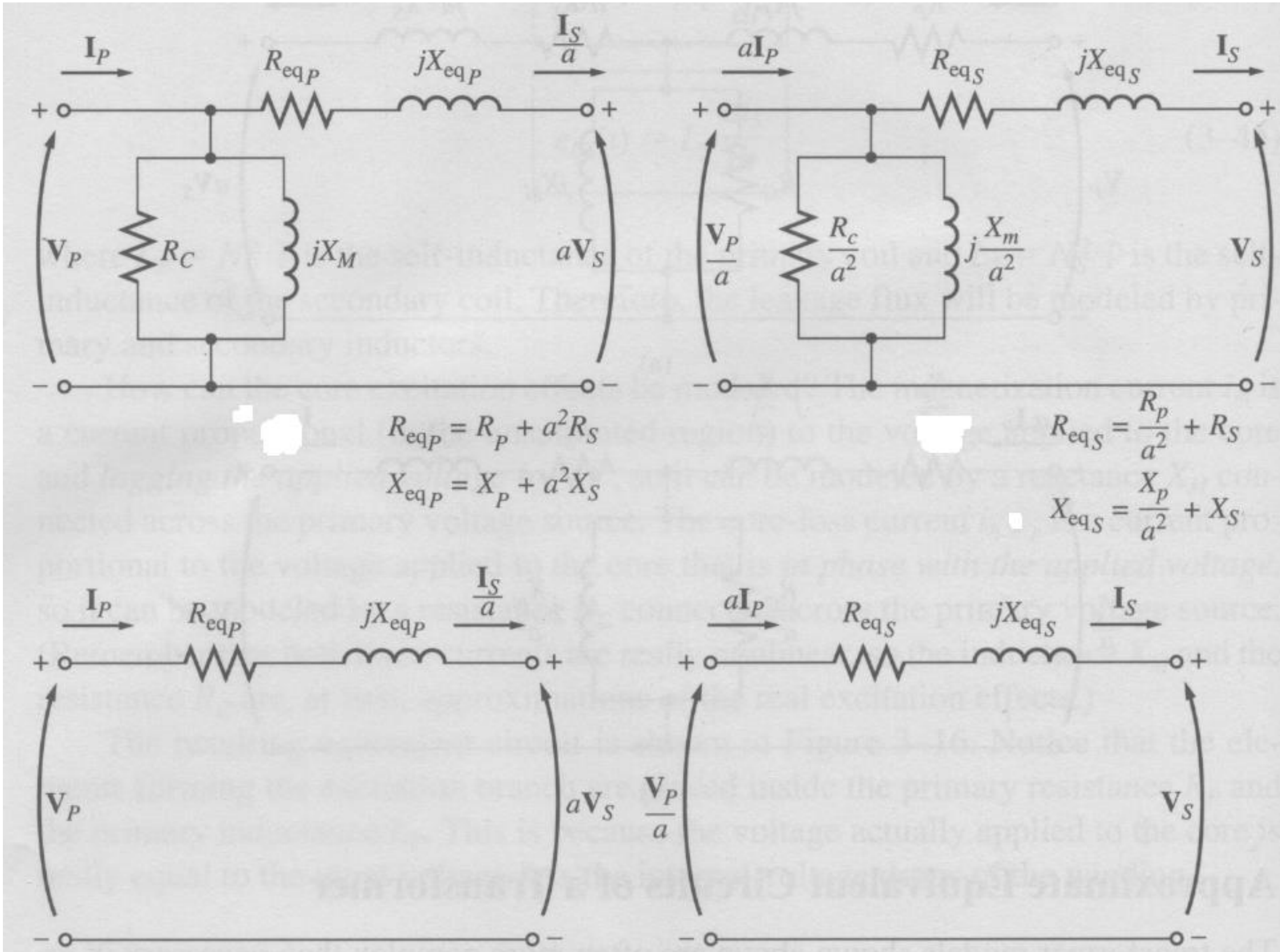
Equivalent circuit of the transformer referred to its primary side.

Equivalent circuit of the transformer referred to its secondary side.



Approximate Equivalent Circuit of a Real Transformer

Referred to the primary side   Referred to the secondary side



Without an excitation branch referred to the primary side.   Without an excitation branch referred to the secondary side.

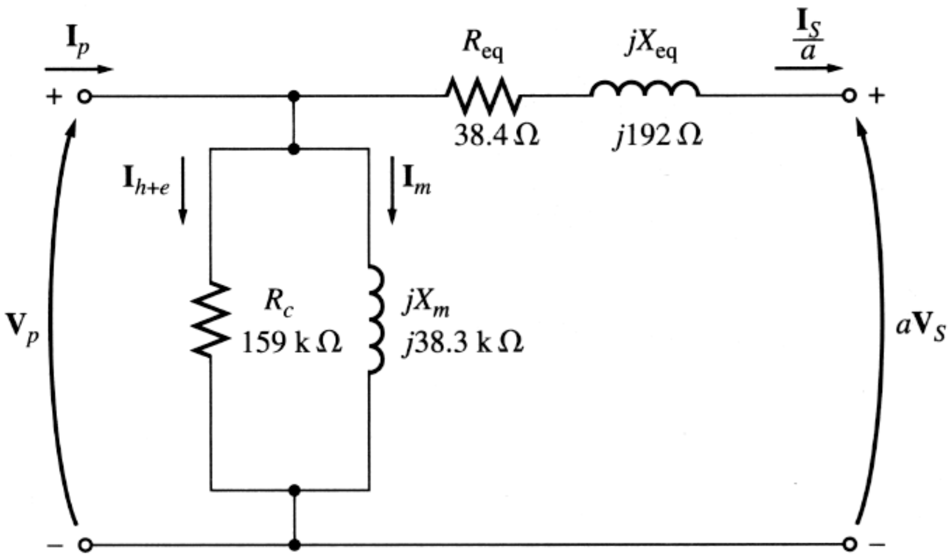
Example

Example 4.2: Determine the equivalent circuit impedances of a 20 kVA, 8000/240 V, 60 Hz transformer. The open-circuit and short-circuit tests led to the following data:

$V_{OC} = 8000\ V$	$V_{SC} = 489\ V$
$I_{OC} = 0.214\ A$	$I_{SC} = 2.5\ A$
$P_{OC} = 400\ W$	$P_{SC} = 240\ W$

$$R_C = \frac{1}{0.0000063} = 159\ k\Omega; \quad X_M = \frac{1}{0.0000261} = 38.3\ k\Omega$$

$$R_{eq} = 38.3\ \Omega; \quad X_{eq} = 192\ \Omega$$



The per-unit system

The voltages, currents, powers, impedances, and other electrical quantities are measured as fractions of some base level instead of conventional units.

$$Quantity\ per\ unit = \frac{actual\ value}{base\ value\ of\ quantity}$$

Usually, two base quantities are selected to define a given per-unit system. Often, such quantities are voltage and apparent power . In a single-phase circuits:

$$P_{base},\ Q_{base},\ or\ \ S_{base} = V_{base} I_{base}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{(V_{base})^2}{S_{base}} \qquad Y_{base} = \frac{I_{base}}{V_{base}}$$

The per-unit system: Example

Example 4.4: Sketch the appropriate per-unit equivalent circuit for the 8000/240 V, 60 Hz, 20 kVA transformer with R_c = 159 kΩ, X_M = 38.4 kΩ, R_{eq} = 38.3 Ω, X_{eq} = 192 Ω.

To convert the transformer to per-unit system, the primary circuit base impedance needs to be found.

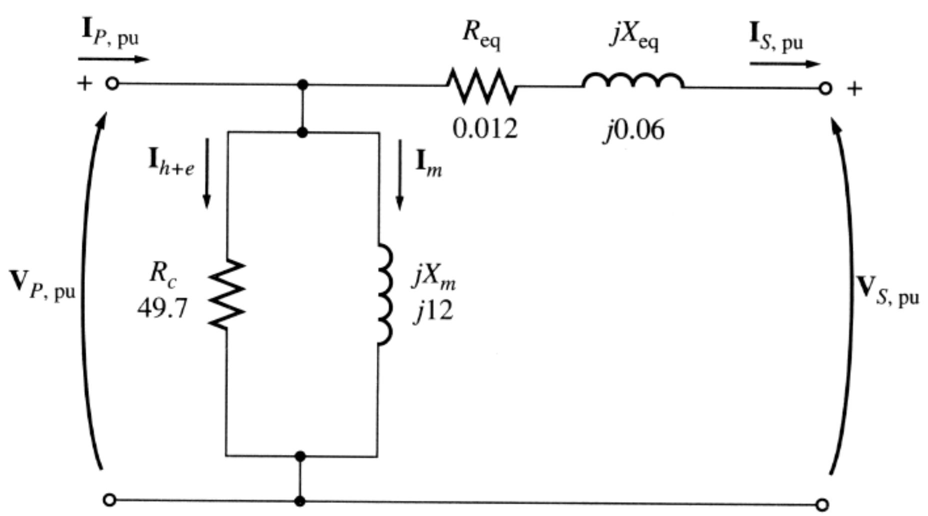
$$V_{base1} = 8\,000\,V; \quad S_{base1} = 20\,000\,VA$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base1}} = \frac{8\,000^2}{20\,000} = 3\,200\,\Omega$$

$$Z_{SE,pu} = \frac{38.4 + j192}{3\,200} = 0.012 + j0.06\ pu$$

$$R_{C,pu} = \frac{159\,000}{3\,200} = 49.7\ pu$$

$$X_{M,pu} = \frac{38\,400}{3\,200} = 12\ pu$$



Voltage Regulation (VR)

Since a real transformer contains series impedances, the transformer's output voltage varies with the load even if the input voltage is constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined as follows:

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s,fl}} \cdot 100\% = \frac{V_p / a - V_{s,fl}}{V_{s,fl}} \cdot 100\%$$

In a per-unit system:

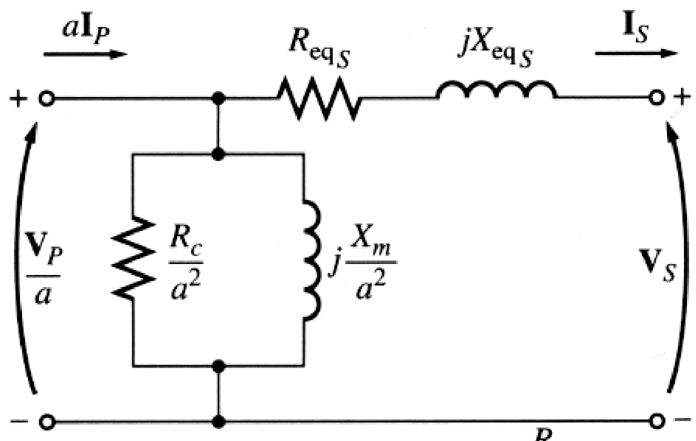
$$VR = \frac{V_{p,pu} - V_{s,fl,pu}}{V_{s,fl,pu}} \cdot 100\%$$

Where $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages.

Transformer Phasor Diagram

Usually, the effects of the excitation branch on transformer VR can be ignored and, therefore, only the series impedances need to be considered. The VR depends on the magnitude of the impedances and on the current phase angle.

$$\frac{V_p}{a} = V_s + R_{eq} I_s + jX_{eq} I_s$$



$$R_{eq\,S} = \frac{R_p}{a^2} + R_S$$

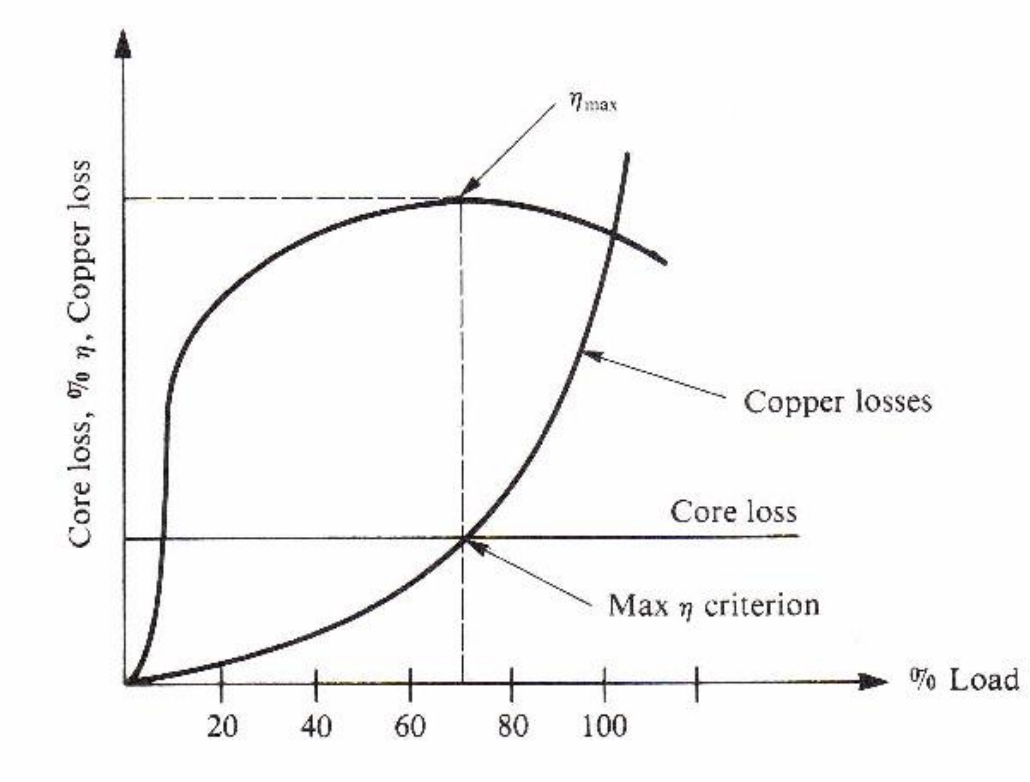
$$X_{eq\,S} = \frac{X_p}{a^2} + X_S$$

Transformer Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \cdot 100\%$$

$$\eta = \frac{V_s I_s \cos\theta}{P_{Cu} + P_{core} + V_s I_s \cos\theta} \cdot 100\%$$

1. Copper (I^2R) losses – are accounted for by the series resistance R_s
2. Hysteresis and eddy current losses are accounted for by R_c.



Transformer efficiency: Example

Example 4.5: A 15 kVA, 2300/230 V transformer was tested to by open-circuit and closed-circuit tests. The following data was obtained:

$V_{OC} = 2300\ V$	$V_{SC} = 47\ V$
$I_{OC} = 0.21\ A$	$I_{SC} = 6.0\ A$
$P_{OC} = 50\ W$	$P_{SC} = 160\ W$

a) Find the equivalent circuit of this transformer referred to the high-voltage side.

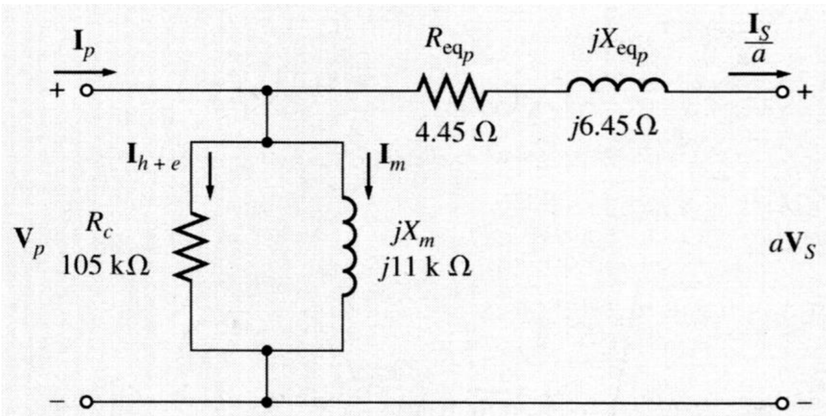
b) Find the equivalent circuit of this transformer referred to the low-voltage side.

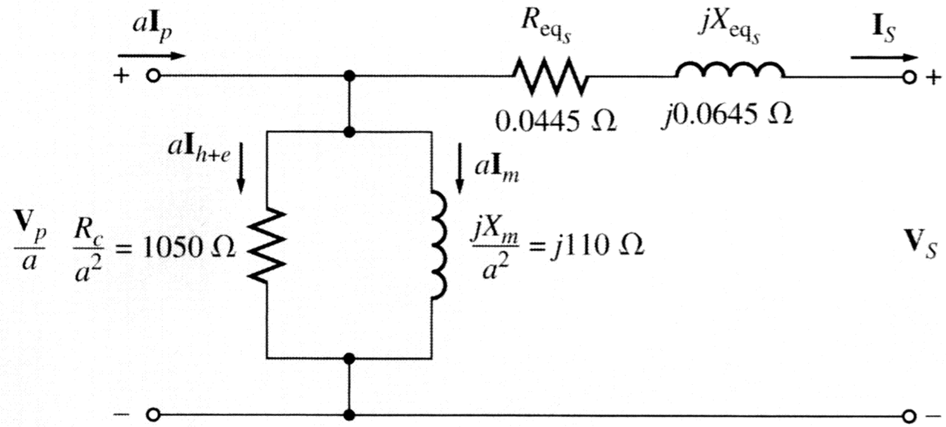
c) Calculate the full-load voltage regulation at 0.8 lagging power factor, at 1.0 power factor, and at 0.8 leading power factor.

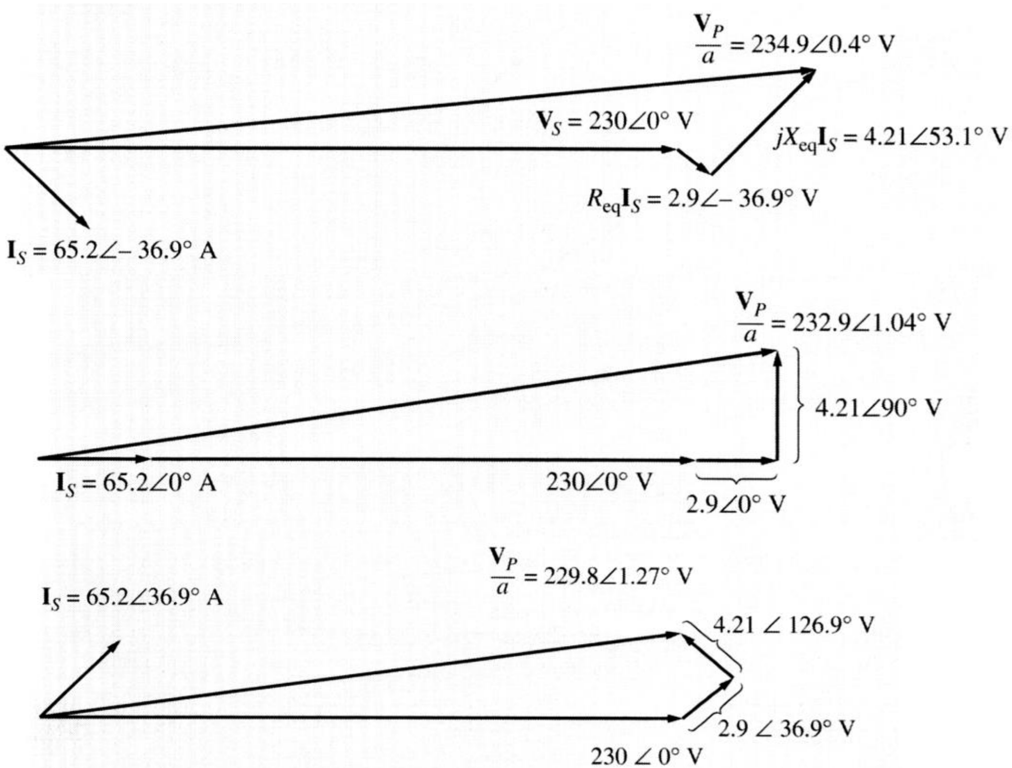
d) Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.

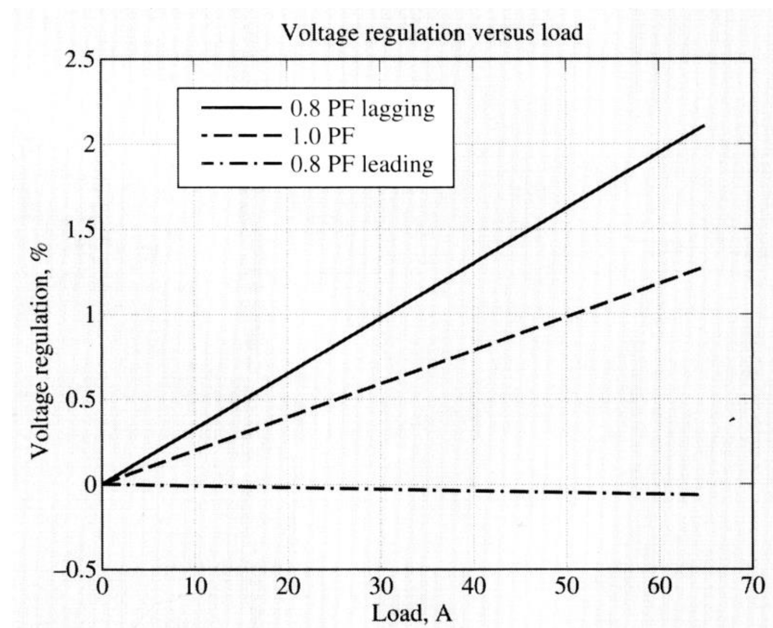
e) What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

Transformer efficiency: Example





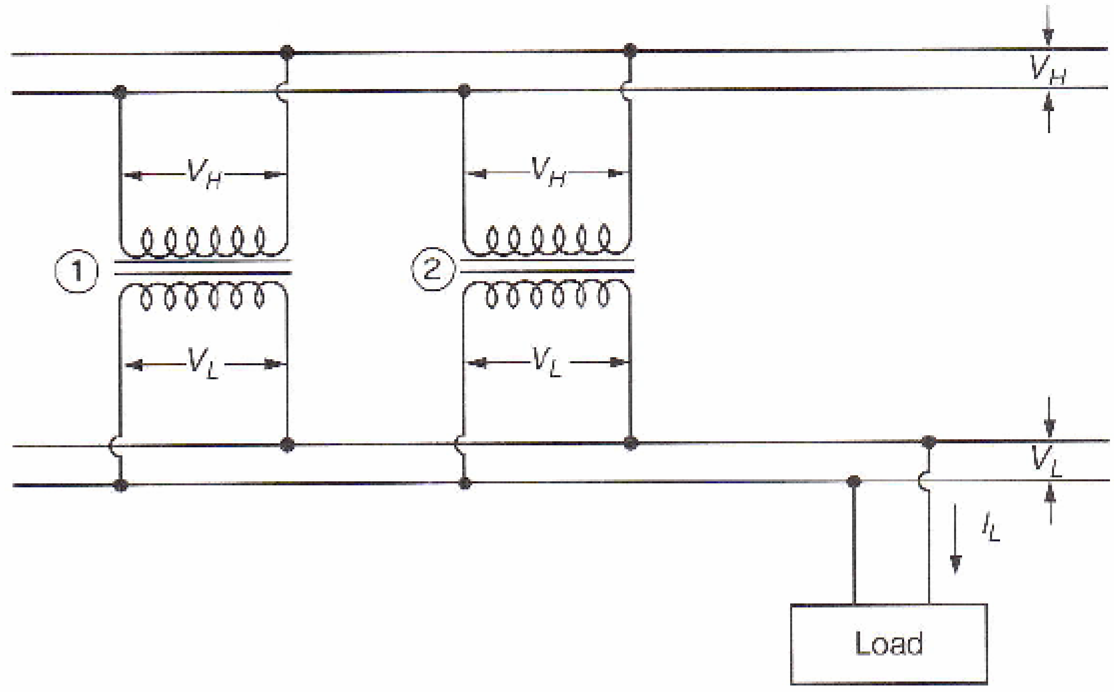


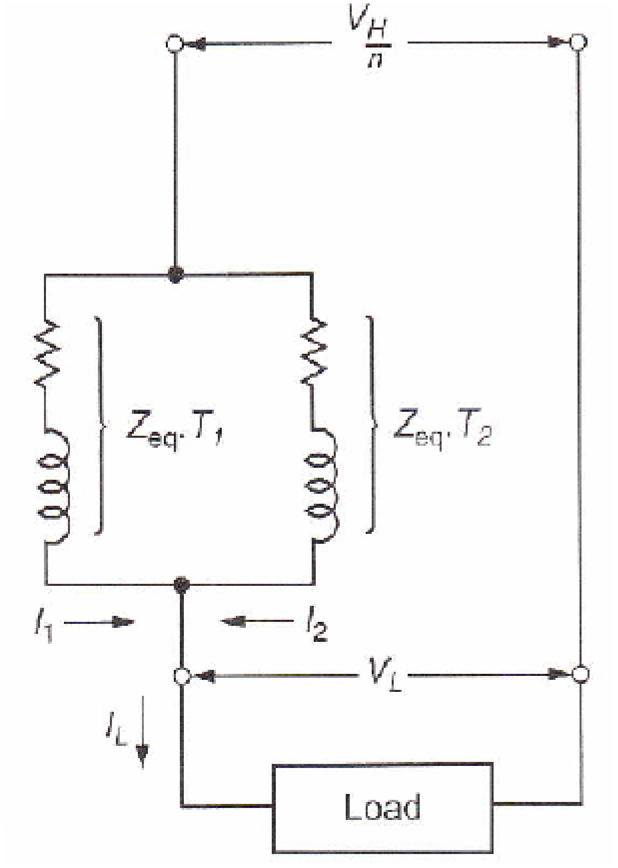


$$\eta = \frac{P_{out}}{P_{Cu} + P_{core} + P_{out}} \cdot 100\% = 98.03\%$$

Transformer Paralleling

Loading Ratio: $$\frac{S_{L1}}{S_{L2}} = \frac{(\%Z)_{T2} S_{T1}}{(\%Z)_{T1} S_{T2}}$$



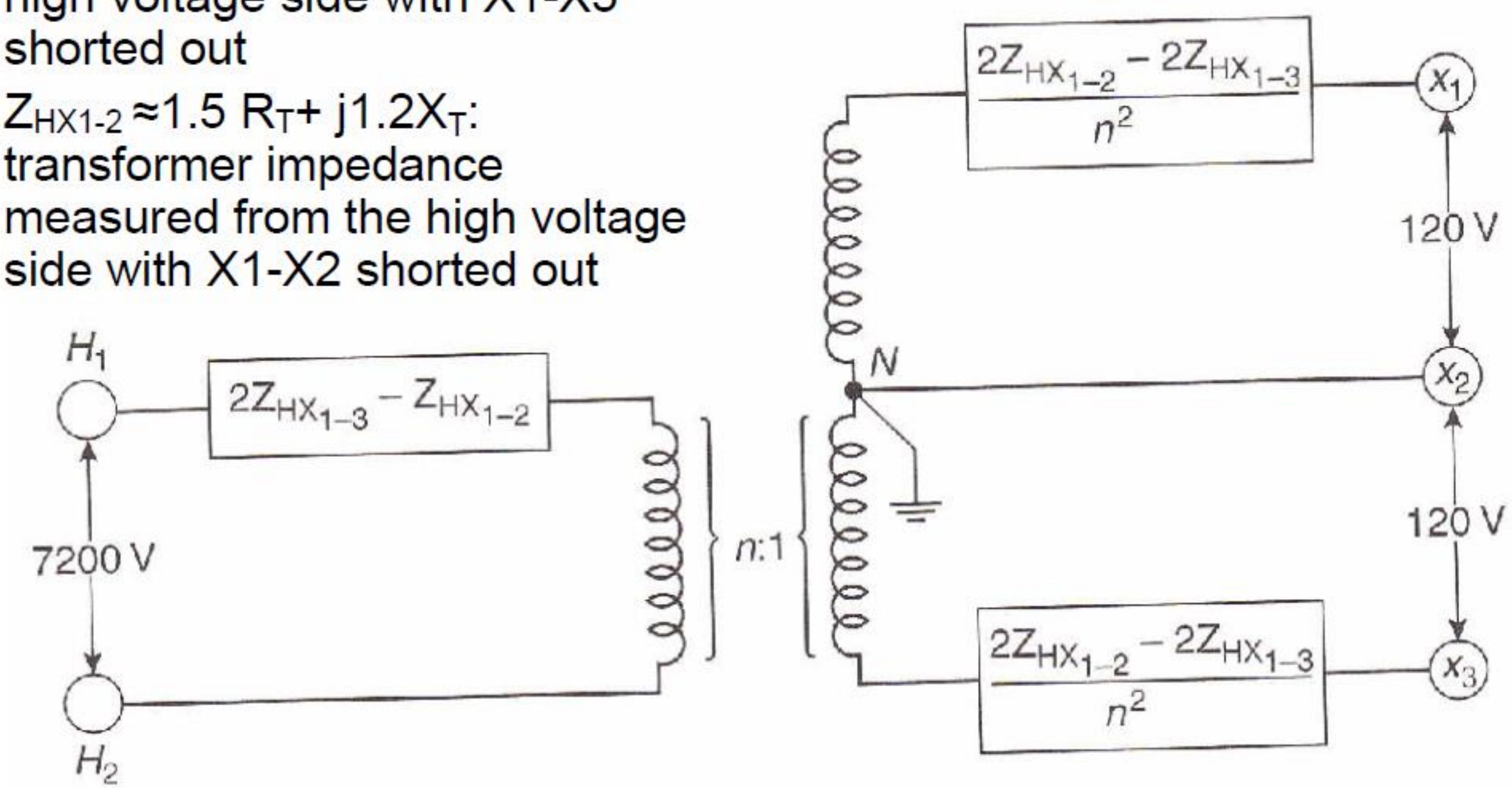


Transformer Paralleling - Example

- 100 kVA transformer is connected in parallel with a 250 kVA. The two transformers are rated at 7,200V/240V, but the 100 kVA unit has an impedance of 4% and the 250 kVA unit has an impedance of 6%. Calculate the following:
  - Nominal primary current of each transformer (ans: 34.7 A and 13.9 A)
  - The impedance of each transformer referred to the primary side (ans: 12.4 and 20.7 Ohms).
  - Loading ratio (ans. 0.6)
  - If the loading on the100 kVA transformer is 100 kVA, then the loading on the 250 kVA transformer is (ans. 166.67 kVA)
  - If the loading on the 250 kVA transformer is 250 kVA, then the loading on the 100 kVA transformer is (ans. 150 kVA)
  - Calculate the rating of the parallel transformer combination (Ans. 266.67 kVA)
  - Assume the transformers supply a load of 330 kVA. Calculate the following:
    - The impedance of the load referred to the primary side (ans: 157 Ohms)
    - The actual primary current in each transformer (ans: 28.8 A and 17.2A)
    - The loading on each transformer (ans.123.75 kVA and 206.25 kVA)

1-Phase Transformer with 3-wire Secondary

- Shunt impedance ignored
- $Z_{HX1-3} = R_T + jX_T$: transformer impedance measured from the high voltage side with X1-X3 shorted out
- $Z_{HX1-2} \approx 1.5\ R_T + j1.2X_T$: transformer impedance measured from the high voltage side with X1-X2 shorted out



Example

- A 25 kVA, 7200/120/240 transformer with Z_{HX1-3}= 0.014 + j 0.012 pu.
  1. Determine the short-circuit current between X1 and X2
  2. Determine the short-circuit current between X1 and X3
  3. Determine the value of the service drop cable resistance (ignore cable reactance) that will result in equal short-circuit currents above.
  4. Determine the corresponding length of a #1/0 AWG aluminum conductor (the resistance of this cable is 4.886×10^{-4} Ω/ft)

- Answer (primary impedance: $24.64\angle 54^{o}$ Ω, secondary impedance: $8.5\angle 20^{o}$ mΩ)
  1. 8,181 A and 136.4 A
  2. 5,649 A and 188.3 A
  3. R = 7.5 mΩ
  4. 38.45 feet

Transformer ratings: Voltage and Frequency

The **voltage** rating is a) used to protect the winding insulation from breakdown; b) related to the magnetization current of the transformer (more important)

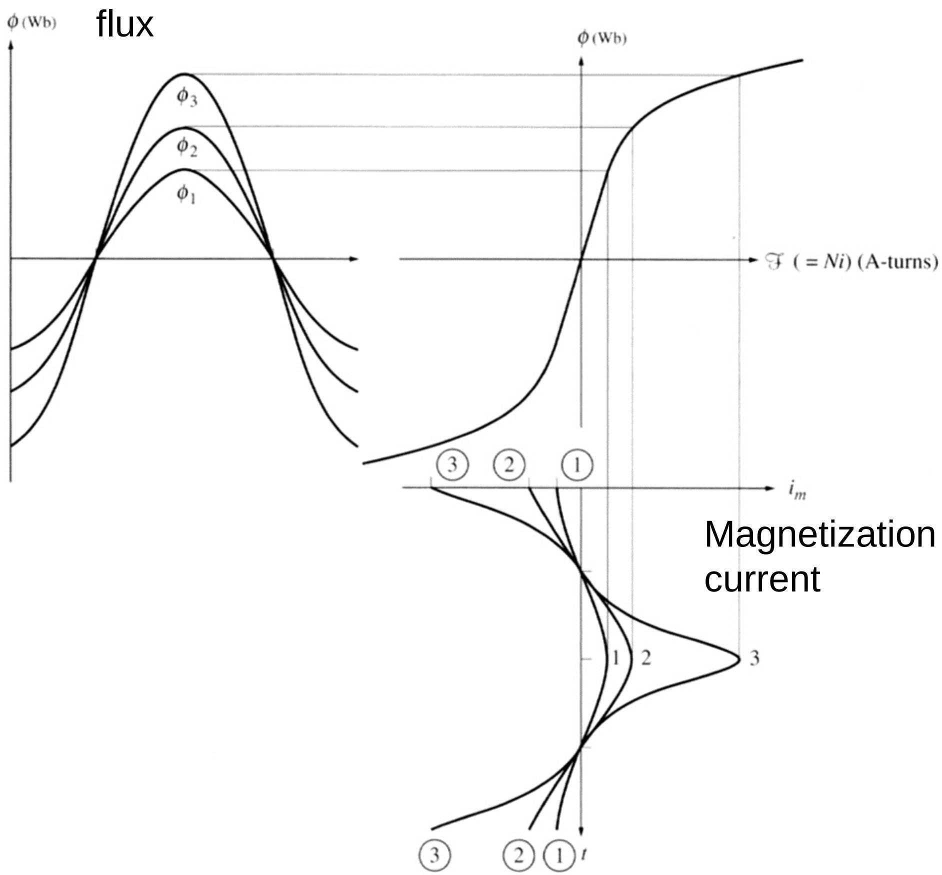
If a steady-state voltage

$$v(t) = V_M \sin \omega t$$

is applied to the transformer's primary winding, the transformer's flux will be

$$\phi(t) = \frac{1}{N_p}\int v(t)dt = -\frac{V_M}{\omega N_p}\cos \omega t$$

An increase in voltage will lead to a proportional increase in flux. However, after some point (in a saturation region), such increase in flux would require an unacceptable increase in magnetization current!



Transformer ratings: Voltage and Frequency

Therefore, the maximum applied voltage (and thus the rated voltage) is set by the maximum acceptable magnetization current in the core.

We notice that the maximum flux is also related to the frequency:

$$\phi_{\max} = \frac{V_{\max}}{\omega N_p}$$

Therefore, to maintain the same maximum flux, a change in frequency (say, 50 Hz instead of 60 Hz) must be accompanied by the corresponding correction in the maximum allowed voltage. This reduction in applied voltage with frequency is called derating. As a result, a 50 Hz transformer may be operated at a 20% higher voltage on 60 Hz if this would not cause insulation damage.

Transformer ratings: Apparent Power

The apparent power rating sets (together with the voltage rating) the current through the windings. The current determines the i^2R losses and, therefore, the heating of the coils. Remember, overheating shortens the life of transformer's insulation!

In addition to apparent power rating for the transformer itself, additional higher rating(s) may be specified if a forced cooling is used. Under any circumstances, the temperature of the windings must be limited.

Note, that if the transformer's voltage is reduced (for instance, the transformer is working at a lower frequency), the apparent power rating must be reduced by an equal amount to maintain the constant current.

Transformer ratings: Current inrush

Assuming that the following voltage is applied to the transformer at the moment it is connected to the line:

$$v(t) = V_M \sin\left(\omega t + \theta\right)$$

The maximum flux reached on the first half-cycle depends on the phase of the voltage at the instant the voltage is applied. If the initial voltage is

$$v(t) = V_M \sin\left(\omega t + 90°\right) = V_M \cos \omega t$$

and the initial flux in the core is zero, the maximum flux during the first half-cycle is equals to the maximum steady-state flux (which is ok):

$$\phi_{\max} = \frac{V_M}{\omega N_p}$$

However, if the voltage's initial phase is zero, i.e.

$$v(t) = V_M \sin\left(\omega t\right)$$

Transformer ratings: Current inrush

the maximum flux during the first half-cycle will be

$$\phi_{\max} = \frac{1}{N_p}\int_0^{\pi/\omega} V_M \sin(\omega t)\,dt = -\frac{V_M}{\omega N_p}\cos(\omega t)\Big|_0^{\pi/\omega} = \frac{2V_M}{\omega N_p}$$

Which is twice higher than a normal steady-state flux!

Doubling the maximum flux in the core can bring the core in a saturation and, therefore, may result in a huge magnetization current!

Normally, the voltage phase angle cannot be controlled. As a result, a large inrush current is possible during the first several cycles after the transformer is turned ON.

The transformer and the power system must be able to handle these currents.